Model Predictive Control of Vanadium Flow Batteries for Optimised Economic Benefit During Power Arbitrage

Shail Godiwala, Lai Wei, Jie Bao, Maria Skyllas-Kazacos

School of Chemical Engineering, University of New South Wales, Australia

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Introduction

- Efficient power arbitrage requires real-time control algorithms
  - Charging/discharging current
  - Electrolyte flow rate

- To make profit from the dynamics of electricity prices
  - Optimise: charging/discharging current, flow rate, and the future SOC profile
  - Electricity price prediction is never accurate → Receding horizon optimisation

- Existing power arbitrage algorithms for VFBs do not consider
  - Vanadium ion diffusion across the membrane
  - Concentration overpotential
  - Flow rate
  - Discharge current > Nominal current (during electricity price spikes)

- A novel power arbitrage algorithm that takes into account these effects has been developed
  - Detailed electrochemical dynamics
  - Detailed hydraulic dynamics

VFB Model: Electrochemical Dynamics

State-space Model of VFB Stack [1]:

$$\frac{V_{stack}}{2}\frac{dc_2^s}{dt} = Q(c_2^t - c_2^s) + \frac{NI}{zF} - N\frac{k_2}{d}c_2^sS - 2N\frac{k_5}{d}c_5^sS - N\frac{k_4}{d}c_4^sS$$
$$\frac{V_{stack}}{2}\frac{dc_3^s}{dt} = Q(c_3^t - c_3^s) - \frac{NI}{zF} - N\frac{k_3}{d}c_3^sS + 3N\frac{k_5}{d}c_5^sS + 2N\frac{k_4}{d}c_4^sS$$
$$\frac{V_{stack}}{2}\frac{dc_4^s}{dt} = Q(c_4^t - c_4^s) - \frac{NI}{zF} - N\frac{k_4}{d}c_4^sS + 3N\frac{k_2}{d}c_2^sS + 2N\frac{k_3}{d}c_3^sS$$
$$\frac{V_{stack}}{2}\frac{dc_5^s}{dt} = Q(c_5^t - c_5^s) + \frac{NI}{zF} - N\frac{k_5}{d}c_5^sS - 2N\frac{k_3}{d}c_3^sS - N\frac{k_3}{d}c_3^sS$$

For stack

Dynamic mass balance equations

For tank

State-space Model of VFB Tank [1]:

$$V_n\frac{dc_2^t}{dt} = Q(c_2^s - c_2^t)$$
$$V_n\frac{dc_3^t}{dt} = Q(c_3^s - c_3^t)$$
$$V_p\frac{dc_4^t}{dt} = Q(c_4^s - c_4^t)$$
$$V_p\frac{dc_5^t}{dt} = Q(c_5^s - c_5^t)$$

SOC of Both Half-Cells:

$$SOC^- = \frac{c_2^t}{c_2^t + c_3^t}$$
$$SOC^+ = \frac{c_5^t}{c_4^t + c_5^t}$$

VFB Cell/Stack Voltage [2]:

$$V_{cell} = E_0' + \frac{RT}{zF}\ln\left(\frac{c_2^sc_5^s}{c_3^sc_4^s}\right)$$

$$V_{stack} = N\{V_{cell} + \mathrm{sign}(I)[|\eta_{con}^-| + |\eta_{con}^+|] + IR_{cell}^{\gamma}\}$$

$$P_b = IV_{stack}$$

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[1] A. Tang, J. Bao and M. Skyllas-Kazacos, “Thermal modelling of battery configuration and self-discharge reactions in vanadium redox flow battery,” Journal of Power Sources, vol. 216, pp. 489-501, 2012

[2] A. Tang, J. Bao and M. Skyllas-Kazacos, “Studies on pressure losses and flow rate optimization in vanadium redox flow battery,” Journal of Power Sources, vol. 248, pp. 154-162, 2014.

VFB Model: Hydraulic Dynamics[3]

Pressure drop in the pipe

$$\Delta p_{pp} = \frac{64}{Re_{pp}} \frac{L_{pp}}{d_{pp}} \frac{\rho Q^2}{2A_{pp}^2} + f \frac{\rho Q^2}{2A_{pp}^2}$$

Pressure drop in the channel

$$\Delta p_{ch} = \frac{64}{Re_{ch}} \frac{L_{ch}}{d_{ch}} \frac{\rho \left(\frac{Q}{N}\right)^2}{2A_{ch}^2} + f \frac{\rho \left(\frac{Q}{N}\right)^2}{2A_{ch}^2}$$

Pressure drop in the porous electrode

$$\Delta p_{pe} = \frac{\mu H_{pe} \left(\frac{Q}{N}\right)}{\kappa L_{pe} W_{pe}}$$

$$\kappa = \frac{d_{pe}^2}{16\lambda_{KC}} \frac{\varepsilon^3}{(1-\varepsilon)^2}$$

$$\Delta p = \Delta p_{pp} + \Delta p_{ch} + \Delta p_{pe}$$

Pump power for each half-cell

$$P_p = \frac{\Delta p Q}{\eta}$$

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[3] Y. Li, J. Bao and M. Skyllas-Kazacos, “Studies on optimal charging conditions for vanadium redox flow batteries,” Journal of Energy Storage, vol. 11, pp. 191-199, 2017.

Economic MPC Formulation: Multivariable Control

- Maximising the economic benefit (the revenue of power arbitrage – operational cost)
- Simultaneous optimisation of current and flow rate → Efficient utilisation of VFB's limited energy capacity
- SOC optimization over a typical electricity price cycle → 24-hour prediction horizon
- Uncertainty in electricity price prediction and process model → Receding horizons
- **Decision variables:** Current and flow rate
- Different timescales for stack/cell voltage and SOC dynamics make this optimisation hard to solve

Optimisation Formulation:

Predicted Operating Cost of VFB

$$\min_{I,Q\,\epsilon\,S(\Delta)} J = \boxed{\int_{t_k}^{t_{k+\alpha}} S_p(t)[P_b(t) + 2P_p(t)]dt}$$

s.t. Electrochemical Model of VFB

Hydraulic Model of VFB

$$I_{min} \le I(t) \le I_{max}$$

$$\dot{I}_{min} \le \dot{I}(t) \le \dot{I}_{max}$$

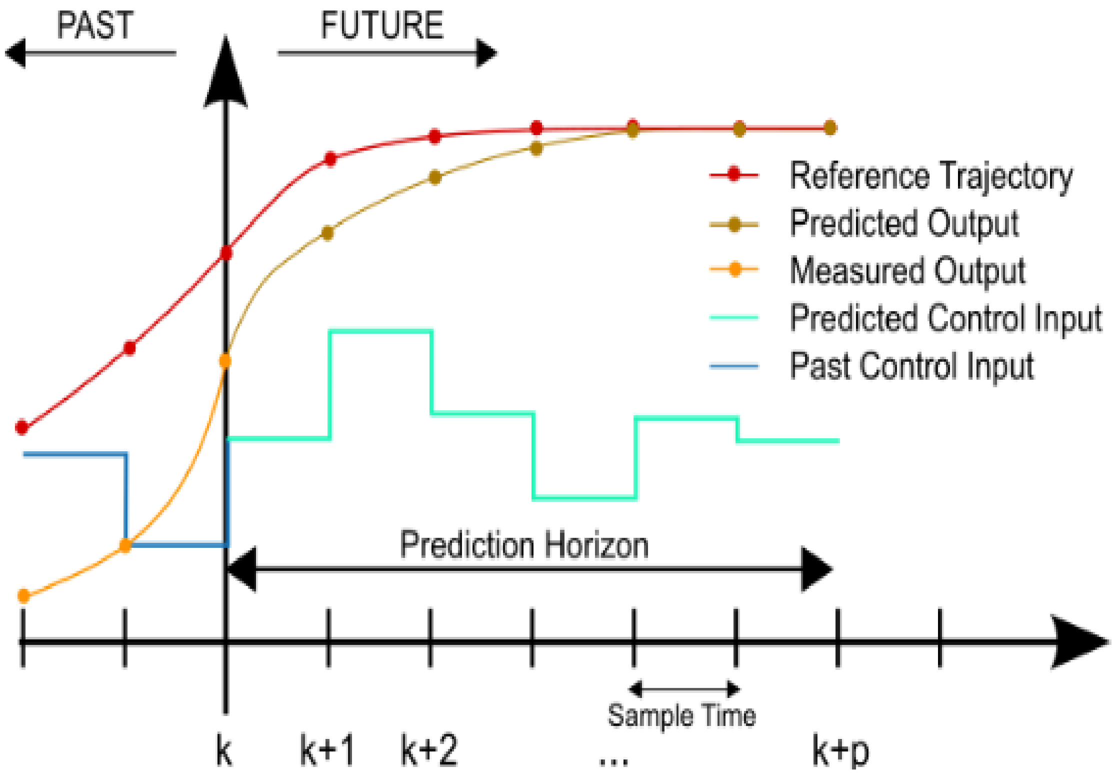
$$Q_{min} \le Q(t) \le Q_{max}$$

$$\dot{Q}_{min} \le \dot{Q}(t) \le Q_{max}$$

$$SOC_{min} \le SOC(t) \le SOC_{max}$$

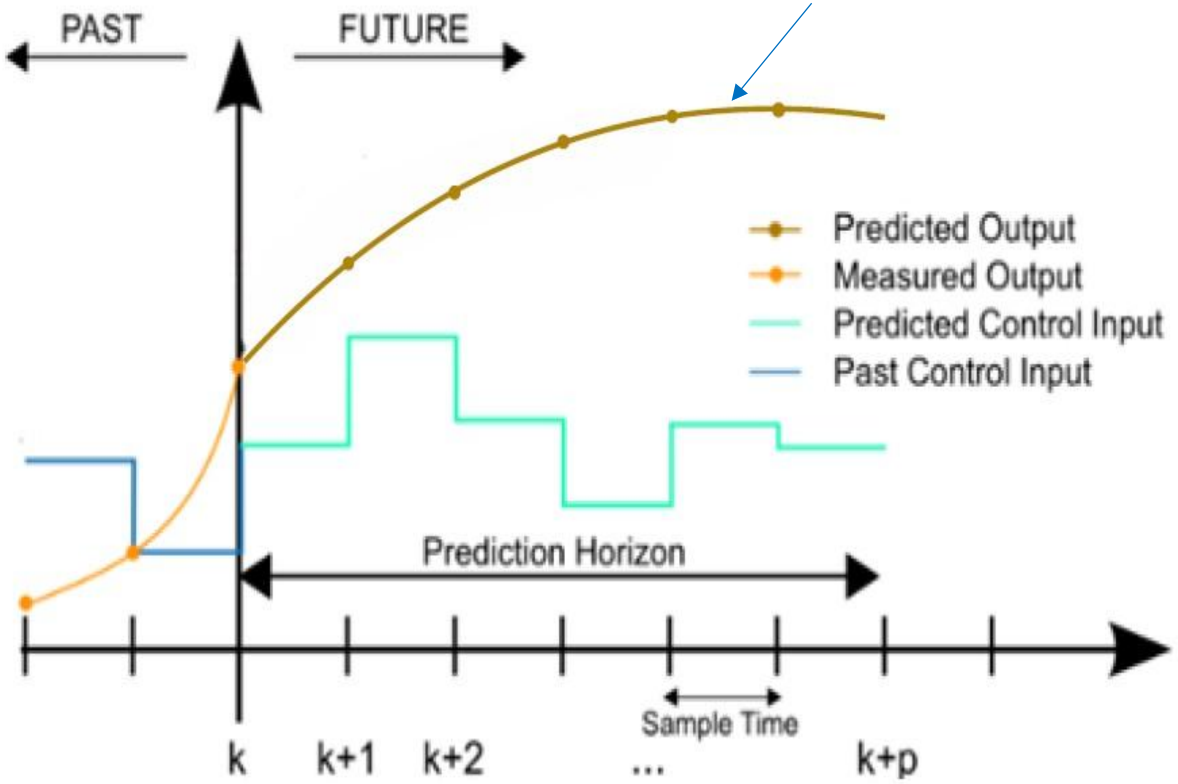
$$V_{s,min} \le V_s(t) \le V_{s,max}$$

Receding Horizon Optimisation



Receding horizon optimisation for MPC
(By Martin Behrendt, via Wikimedia Commons)

Optimal trajectory
(optimised for
economic cost)



Receding horizon optimisation for Economic MPC
(Adapted from Martin Behrendt, via Wikimedia Commons)

Economic MPC Formulation: Bilevel Control – Upper Level

- Only charging/discharging current is optimised with a variable flowrate based on Faraday's law
- Optimisation over 24 hours to efficiently utilize VRB's storage
- Primary focus on SOC dynamics

Predicted Operating Cost of VFB

Optimisation Formulation:

$$\min_{I \,\epsilon\, S(\Delta)} J = \boxed{\int_{t_k}^{t_{k+\alpha}} S_p(t)[P_b(t) + 2P_p(t)]dt}$$

s.t. Electrochemical Model of VFB

Hydraulic Model of VFB

$$I_{min} \leq I(t) \leq I_{max}$$
$$\dot{I}_{min} \leq \dot{I}(t) \leq \dot{I}_{max}$$
$$Q_{min} \leq Q(t) \leq Q_{max}$$
$$SOC_{min} \leq SOC(t) \leq SOC_{max}$$
$$V_{s,min} \leq V_s(t) \leq V_{s,max}$$

Economic MPC Formulation: Bilevel Control – Lower Level

- Given the optimal charging/discharging current, electrolyte flow rate is optimised
- Typically optimized over shorter time periods
- Primary focus on stack voltage dynamics

Predicted Operating Cost of VFB

Optimisation Formulation:

$$\min_{Q\,\epsilon\,S(\Delta)} J = \boxed{\int_{t_q=t_k}^{t_{q+\beta}} S_p(t)[P_b(t) + 2P_p(t)]dt}$$

s.t. Electrochemical Model of VFB

Hydraulic Model of VFB

$$Q_{min} \leq Q(t) \leq Q_{max}$$

$$\dot{Q}_{min} \leq \dot{Q}(t) \leq Q_{max}$$

$$SOC_{min} \leq SOC(t) \leq SOC_{max}$$

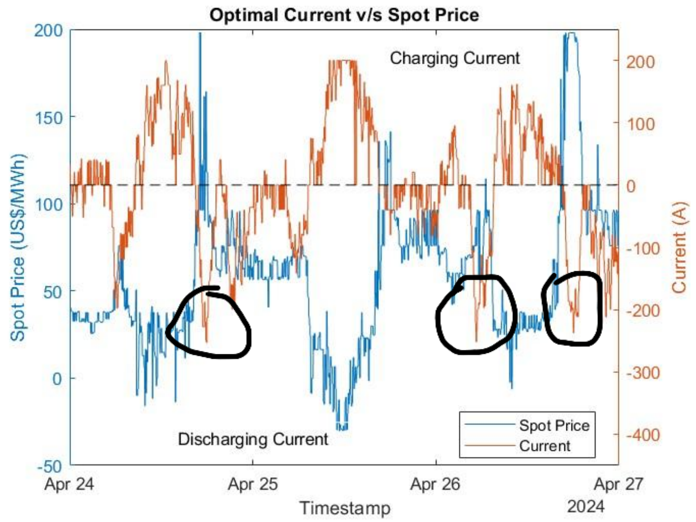
$$V_{s,min} \leq V_s(t) \leq V_{s,max}$$

Parameters for Economic MPC Simulation

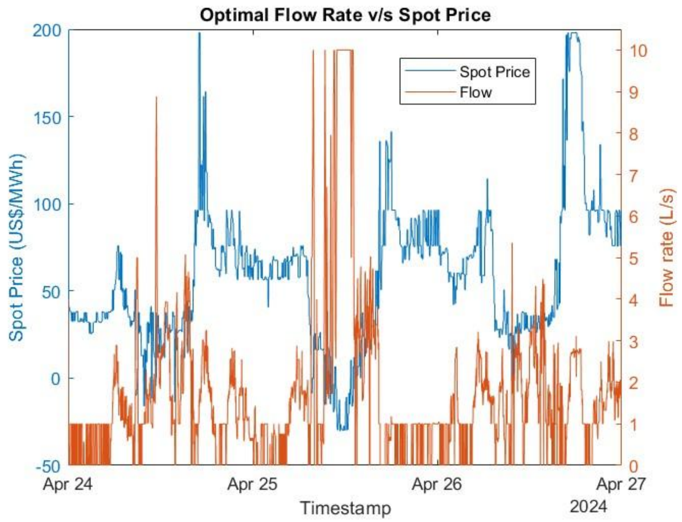
- **VFB Nominal Power:** 52 kW
- **VFB Discharge Period:** 6h
- **VFB Nominal Current:** 200 A
- **VFB SOC Range:** 10 – 90%
- **VFB Stack Voltage Range:** 160 – 360V
- **VFB Semi-permeable Membrane:** FAP-450
- **Electricity Price Profile:** 24-26 April 2024 for the State of New South Wales, Australia [4]
- **Prediction Horizons:**
  - Upper-level: 24 hours
  - Lower-level: 5 minutes

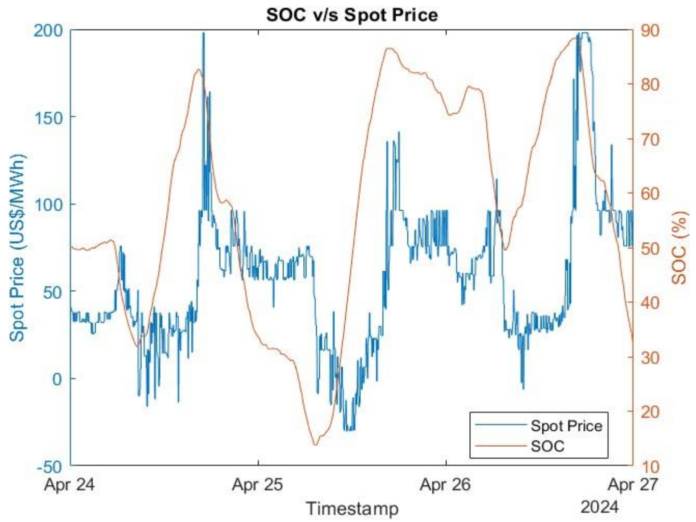
[4] https://aemo.com.au/en

Bilevel Economic MPC Simulation Results

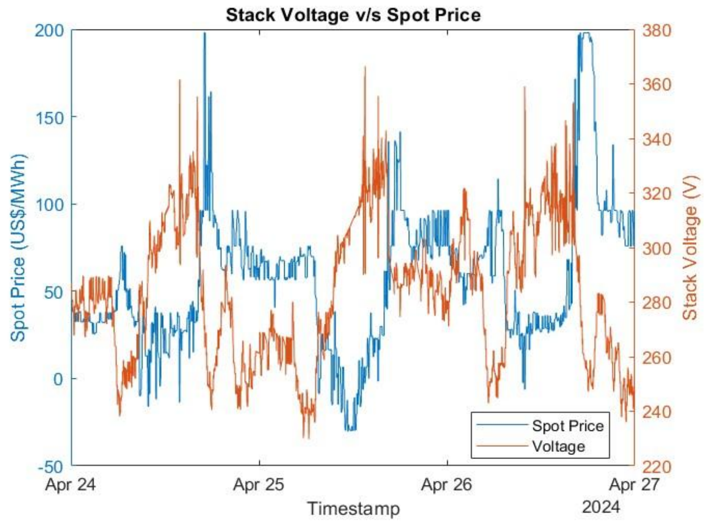


Primary Objective:
Operating revenue maximisation/cost minimisation





Secondary Objective:
Efficiency maximisation



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Performance Comparison: Bilevel Economic MPC v/s Fixed-Time Charging/Discharging

3-Day Efficiency (Optimized for Revenue)	Fixed-Time Charging/Discharging Operation		Bilevel Economic MPC
	Constant Flow	Variable Flow	
Coulombic Efficiency (%)	97.11	97.10	**96.18**
Voltage Efficiency (%)	87.54	83.04	**82.04**
Energy Efficiency (%)	85.01	80.63	**78.91**
System Efficiency (%)	66.22	77.99	**73.09**

3-Day Revenue	Fixed-Time Charging/Discharging Operation		Bilevel Economic MPC
	Constant Flow	Variable Flow	
Gross Revenue (US$)	25.31	22.40	**55.41**
Pumping Cost (US$)	13.36	1.80	**0.87**
Net Revenue (US$)	11.95	20.60	**54.54**

Simple Model-based Control Approach for VFBs

Simple VFB Model [5]:

$$\frac{d(SOC)}{dt} = \begin{cases} \dfrac{P\eta_{charge}}{E_m} & \text{when } P > 0 \\ \dfrac{P}{E_m \eta_{discharge}} & \text{when } P < 0 \end{cases}$$

Optimisation Formulation:

$$\min_{P \,\epsilon\, S(\Delta)} J = \int_{t_k}^{t_{k+\alpha}} S_p(t)P(t)\,dt$$

s.t. Simple Model of VFB

$$P_{min} \leq P(t) \leq P_{max}$$

$$SOC_{min} \leq SOC(t) \leq SOC_{max}$$

[5] X. Zhang, J. Bao, R. Wang, C. Zheng and M. Skyllas-Kazacos, “Dissipativity based distributed economic model predictive control for residential microgrids with renewable energy generation and battery energy storage,” Renewable Energy, vol. 100, pp. 18-34, 2017

Performance Comparison: Bilevel Economic MPC v/s Simple Model-based Control

3-Day Efficiency (Optimized for Revenue)	Simple Model Control	Bilevel Economic MPC
Coulombic Efficiency (%)	96.48	**96.18**
Voltage Efficiency (%)	76.27	**82.04**
Energy Efficiency (%)	73.58	**78.91**
System Efficiency (%)	71.21	**73.09**

3-Day Revenue	Simple Model Control	Bilevel Economic MPC
Gross Revenue (US$)	47.33	**55.41**
Pumping Cost (US$)	1.20	**0.87**
Net Revenue (US$)	46.13	**54.54**

Conclusion

- A novel power arbitrage algorithm developed to take into account:
  - Vanadium ion diffusion across the membrane
  - Concentration overpotential
  - Flow rate
  - Discharge current > Nominal current (during electricity price spikes)

- A novel bilevel economic MPC approach developed to reduce the computational burden:
  - Upper-level: Optimisation of current to effectively utilise VFB’s energy storage capacity
  - Lower-level: Optimisation of flow rate to deal with fast-moving stack voltage dynamics

- Upon comparison:
  - Higher revenue generation than that of conventional approaches
  - Superior VFB efficiency than that of a simple control approach

- Future work:
  - Experimental studies
  - Optimisation based on probabilistic distribution of the electricity price prediction error (a stochastic system control approach)